Lithium-Ion Powered Hyperloop*

Sarah Munday
Julia Win
Taylor Boelkes

Abstract

Over the past several years, the state of California has considered a proposal to build a high-speed rail line connecting San Francisco and Los Angeles, as well as several other smaller locations. As plans have developed, the project has been hampered by expanding costs and questions about its feasibility. Recently, inventor Elon Musk proposed the construction of a new transportation system - a 'Hyperloop' which would use transports suspended in low-pressure steel tubes and powered by external motors to move passengers from one location to another. A key component of the Hyperloop is its onboard energy supply, which powers a turbine needed to suspend the capsule on a low-friction cushion of air[1]. Because space is limited, an efficient and high energy-density storage system is needed to power the capsule. Three possible energy storage systems are being considered: lead-acid batteries, lithium-ion batteries, and a new compressed air system called LightSail. The goal of this report is to provide a final recommendation for the 'best' energy storage system for the onboard Hyperloop turbine through analysis of the strengths and weaknesses of three options. Each of the three options will be evaluated for safety, reliability, energy density, and cost to determine which will be the most suited to the Hyperloop.

Taylor Boelkes, Jonathan Gallien, Maurice Harari, Carson Hughes, Hojin Jeon, Peter Milling, Sarah Munday, Julia Win

Background and Motivation

The following systems are being compared: LightSail's compressed air energy storage, lithium-ion batteries, and lead-acid batteries. Lithium-ion and lead-acid batteries will be investigated due to their reputation as reliable energy storage technologies and LightSail will be investigated due to Elon Musk's recommendation for its use in the Hyperloop. Any storage system would have to be capable of driving an electric motor capable of running the turbine. The driving force on the capsule is not derived from the turbine itself: the turbine will redirect air into spaces underneath the capsule, suspending it on a cushion of air, allowing low friction motion.

LightSail

LightSail is a new technology that is primarily mechanical rather than chemical in nature. It consists of a piston and a water-storage tank. To generate work, hot water is sprayed into the piston containing compressed gas; the gas then expands and does the work needed by the compressor. To recharge, work must be done to compress the gas, releasing heat. The water absorbs the heat and flows into storage. The air always stays around room temperature. LightSail operates with remarkable simplicity compared to the battery alternatives, and the inventors further claim that the LightSail's thermodynamic process is fully

*Version 1.1: Oct 31, 2013 5:33 am -0500
†http://creativecommons.org/licenses/by/3.0/

reversible and thus incredibly efficient[2]. While the creators of LightSail give impressive numbers such as a 20 year lifespan and 90% thermal efficiency, data regarding concerns such as size and safety is not publicly available.

Lithium-ion Batteries

Lithium-ion batteries are made up of two components joined by an electrolyte: a lithium-containing compound and a matrix which can accept intercalated lithium-ions[3]. While a number of materials can be used for cathodes and anodes, the cathodes are typically compounds such as lithium iron phosphate (common in vehicles) or lithium cobalt oxide (common in electronics), while the anode materials vary widely. Energy densities of 0.12 kWh/kg[4] are common. Lithium-ion batteries have lower discharge efficiencies than other battery types, as the electrolyte can degrade at low voltages. In general, lithium-ion batteries

have higher energy densities and shorter lifespans than other battery types, as well as monthly charge loss when not in use.

Lead-Acid Batteries

Lead-acid batteries are made up of lead and lead oxide plates submerged in sulfuric acid. These batteries can typically be run for 1200 recharge-discharge cycles before the plates become too degraded by sulfation (conversion of elemental lead to lead sulfate) to provide useful voltage. The type of battery considered is a 'sealed' lead-acid battery; its contents are not exposed to air. This prevents any hydrogen and oxygen gas produced by electrolysis of the aqueous solution from being exposed to stray sparks or open flames. Lead-acid batteries are relatively maintenance free, and often have safety features incorporated into their designs. However, their specific energy densities tend to be low.

Evaluation Criteria

In order to determine which of the three energy sources is the most appropriate to power the compressor required for the capsules in the Hyperloop system, each of the options was evaluated with respect to five distinct criteria. The five criteria used to evaluate the power sources are the following: energy density, performance/efficiency, cost, reliability, and safety. Each of the criteria was given a weighting under the constraint that the sum of the criteria must be equal to 1, as shown in Figure 1 below.

Criteria	**Weight**	**LightSail**	**Lithium-Ion**	**Lead-Acid**
Energy Density	0.30	1	5	3
Safety	0.25	1	3	3
Cost	0.20	5	1	3
Performance	0.15	4	5	4
Reliability	0.10	2	4	4
Total		2.35	**3.60**	3.25

Table 1

1	**2**	**3**	**4**	**5**
Terrible	Poor	Neutral	Good	Great

Table 2

Figure 1. The Pugh Matrix developed to analyze the three selected energy storage systems with the rating scale.

Energy density was given the highest weighting, 0.3, largely because this is a measurable quantity for each energy source and therefore there is no subjectivity introduced when comparing the energy sources.

Additionally, the compressor in each capsule requires a certain amount of power in order for the trip to be successful, so the energy density of the energy source must meet the requirements of the system.

Safety was given the second highest weighting, 0.25. The Hyperloop system is expected to carry greater than 14 million passengers per year and therefore the safety of the passengers must be of utmost importance. If the patrons are put at risk by the implemented energy storage technology, the entire system cannot be put into operation.

Although there is not a cost constraint introduced in the initial analysis of the Hyperloop system, it is important to consider the initial and maintenance costs of the energy source. Therefore, the cost of the energy storage system was given a weighting of 0.2. When considering the cost of the energy source, the lifespan, cost of materials, cost of repairs, and recharge time were considered. The recharge time was a large portion of the cost consideration because it determines the number of energy sources that must be either in use or charging at any given time.

The performance and efficiency of the energy source was given a weighting of 0.15. Not only must the energy source be able to consistently produce the amount of power required for the compressor, but this efficiency also cannot decrease over the lifespan of the source. Because the energy source will be removed from the capsule at each station and recharged before being used again, the energy source must also be capable of being recharged numerous times without losing efficiency.

The final criteria, reliability, was given a rating of 0.1. In order for the energy source to be a feasible way to power the capsules, the maintenance and ease of repair of the energy sources must be important in the case of failure. However, because the trips are only 45 minutes in duration, the chance of failure in these short trips is low.

Discussion

Assumptions

Several assumptions were made when comparing the energy sources in order to determine which is the most appropriate to power the compressor that is required in each capsule. The first major assumption made is that the power source must only power the compressor. Power requirements in the cabin such as lighting, the entertainment system, or air conditioning were not accounted for, since the power to the compressor is what is completely necessary for the Hyperloop to run. Additionally, it is also assumed that the motor can power the compressor with 100% efficiency. As a result, all of the available energy stored will be converted directly into power for the compressor. Finally, it is assumed that the power required by the compressor does not vary with the weight of the energy storage system. As the operation of the compressor and the conditions of the environment in the Hyperloop are not specified, it is difficult if not impossible to accurately predict the relationship between the power requirement and battery weight.

Lithium-ions are commonly drained to about 60% of their capacity before recharge[5]. This wear-down characteristic can be applied to other batteries of similar chemistries[6]; therefore it is assumed that lead-acid batteries are also drained to this extent. This drainage characteristic is common among many batteries due to an inability to recharge if drained to full capacity. As little information is known on the functionality of LightSail, however, it is assumed that the compressed air can be fully expanded, thus draining 100% of the accessible energy. It is also assumed that the charging capacity and energy density of the batteries does not degrade with use and time. The life-span of the batteries was taken into consideration in the Pugh matrix during analysis. It is assumed that the lifespan of the battery already accounts for any charging capacity and energy density loss.

In the document released by Tesla, Musk states that the weight of the batteries is to be 1,500 kg[1]. However, because Musk does not specify which type of battery this number

corresponds to or the dimensions of this battery, all calculations were computed under the assumption that this weight is a suggestion rather than a constraining factor.

The remainder of the assumptions made are related to the LightSail calculations. The first is that the values obtained for the temperature and pressure used for the compressed air are accurate. As these values are not directly available, values cited in articles on LightSail's technology were used. The second assumption is that the work gained in the LightSail process can be explained by a single, near isothermal and reversible expansion. Although the change in air temperature is greater than zero, it is relatively small (less than

10 °C)[2]. As a result, the calculations were done as though the process is isothermal. The final assumption made is that the Redlich-Kwong equation of state used for the LightSail calculations is applicable. Due to the high pressure of the compressed air (~ 3000 psi), the ideal gas law cannot be applied. The Redlich-Kwong equation, however, can be applied because it takes intermolecular interactions that gases experience at high pressures into consideration.

Energy Storage Type	**Specific Energy Density (Wh/kg)**	**Energy Density (Wh/L)**	**Efficiency (η)**	**Mass Required (kg)**	**Volume Required (L)**	**Cost per Unit ($)**	**Twenty-Year Cost ($ million)**
Lead-Acid	35	70	0.71	16,350	8,200	36,600	316
Lithium-Ion	120	230	0.85	4,000	2,100	168,000	931
LightSail	5.3	13	0.70	>69,600	26,870	139,200	14

Table 3

Figure 2. A summary of the traits and requirements associated with each energy storage system.

Energy Density

The energy densities of the lithium-ion battery are far better than both the lead-acid battery and LightSail. The mass energy density of the lithium-ion battery is 0.12 kWh/kg (or a volumetric energy density of 0.230 kWh/L)[4] as compared to that of the lead-acid battery, which is 0.035 kWh/kg (or a volumetric energy density of 0.0675 kWh/L)[7]. However, in order for LightSail to power the compressor with 243.75 kWh, the technology requires 12.0 m^3 of volume - much more than the lithium-ion and lead-acid options require: 2.08 m^3[D2] and 8.17 m^3[C2] respectively. This also results in a LightSail module weight requirement of approximately 69.6 metric tons to power each capsule[B10] - almost 30 times the weight proposed by Musk[1]. Therefore LightSail has an energy density much lower than that of the lithium-ion and lead-acid battery. A summary of these results is provided in Figure 2.

Safety

For the safety evaluation of the storage options, the lithium-ion and lead-acid batteries have the same safety rating, while LightSail has a much lower one. This is mostly attributed to the lack of information regarding the safety of LightSail. It is unknown how the safety risks involved with LightSail may change in the low-pressure and high-speed conditions it would be subjected to in the Hyperloop. In an isothermal compression, if the water entering the cylinder is greater than the cylinder's minimum volume, water can become trapped in the cylinder. This is known as hydrolocking, and can lead to mechanical damage or an explosion. According to LightSail, this has not been an issue in their technology thus

far[8]. However, it is unknown what the extent of the damage would be if the tank containing the high-pressure air were to explode.

In comparison, lithium-ion and lead-acid batteries have been used safely in many industries, most notably in transportation. There is a risk of explosion if the battery is not properly maintained or is misused, but this risk is relatively small. In the United States, 2,300 people are injured each year from using lead-acid batteries and approximately 50% of these injuries are due to lifting or dropping the battery[9]. Compared to the approximate 250 million registered passenger vehicles in the United States, the risk of explosion is insignificant. There is no evidence to suggest that the safety risks involved in lead-acid battery use are notably higher than other energy storage systems capable of the same performance.

Cost

One of the greatest advantages LightSail has over its competitors is price. Air and water are the only parts of the system that need replenishing. Air is free and water is relatively cheap. The water cost could be

further reduced by using a closed-loop system, which allows for the water to be recycled. This suggests that the main cost of LightSail is the equipment used for compression and storage. LightSail will not disclose the material used for the storage tank, so the cost is calculated assuming the tank is built from steel. LightSail has proposed two sizes for its technology. As its smaller size is comparable to the size required from the calculations, it is assumed that the air can be stored in a single cylindrical tank. Given the dimensions of the Hyperloop, the necessary thickness of steel to withstand a pressure of 3000 psi, and the volume requirement of the tank, the tank should cost about $104,400[E1]. Assuming a total cost of $139,200 per module[E2] and 100 modules in commission, the LightSail cost would be about$14 million[E3].

The cost of a lithium-ion battery that meets the power requirements is estimated to be about $168,000 per battery[E6]. This value was obtained using a cost of $0.689/Wh[10]. Taking into consideration the number of trips made each year and the number of recharge times during the battery's lifetime, the total annual cost of replacing the batteries is $46.52 million[E7]. While lithium may be subject to price shocks based on its availability on the market, the cost of lithium in batteries makes up less than 1% of the total cost of the battery[11]. As a result, any lithium cost fluctuations are fairly negligible.

The cost is estimated to be $36,600 per lead-acid battery[E10]. This value was obtained using a cost of $0.15/Wh[12]. Taking into consideration the number of trips made each year and the number of recharge times during the battery's lifetime, the total annual cost of replacing the batteries is $15.77 million[E11]. This is approximately two-thirds less than the annual cost of using lithium-ion batteries.

Considering the lifetime of LightSail, which claims to be over 20 years, this cost is much lower than the cost of lithium-ion or lead-acid batteries over that same time span (approximately $931 million[E8] and $316 million[E12] respectively - shown in Figure 2).

Performance/Efficiency

Efficiency is the ratio of the work done by the energy source compared to the amount of energy that was stored in it to do that work. LightSail has an overall efficiency of 70% according to the LightSail website[2]. The efficiency of lead-acid batteries varies from 50-92%[13]. It is assumed that this value is 71%, as this is the median efficiency. The efficiency of a lithium-ion battery is around 85%[14]. The efficiencies of LightSail and the lead-acid battery are comparable while the lithium-ion battery has the highest efficiency of the three

energy storage systems. Any self-discharge in the batteries is accounted for in the calculation of the power needed[A1]. These values are summarized in Figure 2.

Reliability

As LightSail is a relatively new technology, it has not been widely used. Due to the lack of information LightSail has made publicly available, any malfunctions with the system may result in unknown damage and significant repair time.

Lithium-ion and lead-acid batteries have been widely used with successful results. Due to the wide variety of applications, specifically in transportation, it is assumed that both batteries will have similar reliabilities in the Hyperloop. Additionally, should the battery fail or malfunction, there is extensive information and experience available to solve the problem quickly and cost-effectively. This results in lithium-ion and lead-acid being ranked the same in reliability, while LightSail is ranked much lower.

Recommendation

Based on the evaluations of each energy storage system in the five categories stated above and the weighting assigned to each category, the best storage system for the Hyperloop is the lithium-ion battery. The Pugh Matrix rankings and weightings produced the following overall scores on a scale from 1 to 5: LightSail - 2.35, lithium-ion - 3.60, lead-acid - 3.25 (see Figure 1). The lithium-ion battery received the highest score in energy density and performance/efficiency. It was tied with the lead-acid battery for the highest score in safety and reliability. The lithium-ion battery received the lowest score of the three in cost. Although the cost of the lithium-ion battery is significantly higher than that of the lead-acid battery and LightSail, its superiority in the remaining categories suggest that it is the best option of the three storage systems analyzed.

Pitfalls, Limitations, Alternatives

Pitfalls

There are a few pitfalls that each type of battery possesses. For example, lithium-ion and lead-acid

batteries degrade each time they are recharged. This means that their energy density is constantly decreasing. The rate of degradation varies by type of batteries, temperature of surroundings, etc. It is hard to accurately predict the degradation rate, yet it can be assumed that for degradable batteries, this has a negative effect on their lifespans. Another pitfall is the self-discharge of batteries. Self-discharge is the phenomenon in which batteries discharge even when they are not in use. While this characteristic may be small, it could cause significant changes in the batteries' lifespans.

A major pitfall of LightSail is that there is little information available to the public. This results in a number of assumptions being made in order to estimate weight, volume, and cost values. It also decreases the confidence with which it can be said that it is safe, reliable, and efficient. As the safety risks are unknown under normal operating conditions, it is difficult to extrapolate any of these risks to the low-pressure, high-speed environment in the Hyperloop.

Limitations

Apart from the pitfalls that each battery type possesses, a number of assumptions were also made in analysis, resulting in some limitations. The most important limitation is that the volume and weight required for each energy storage system is larger than that suggested by Musk. The size and weight of the energy storage systems are heavily weighted

in the analysis. Therefore, the analysis resulted in determining which battery has the most potential to meet these requirements rather than which battery meets or exceeds them.

Alternatives

For subjects of analysis other than LightSail, only those currently being widely used were chosen. Once their safety is established, several batteries currently undergoing research, such as hydrogen fuel cells and molten salt batteries, may be used as alternatives.

Hydrogen stores a huge amount of energy, as much as 32 kWh/kg,[16] in the form of chemical energy. Although its energy reduces in the course of stabilization, it can still reach a .330 kWh/kg energy density with present technology,[17] and research shows that it can even store up to 8 times as much energy as lithium-ion batteries can.[18]

Though available molten salt batteries have energy densities comparable to the lithium-ion battery [19], a new molten salt battery being developed by Sumitomo Electric Industries Ltd. and Kyoto University has achieved energy densities even higher than those of the lithium-ion battery, reaching up to .290 kWh/L. Since the battery is also non-volatile and non-flammable, the batteries can be packed closer together, resulting in a higher energy density[20]. Thus either the hydrogen fuel cell or the molten salt battery may be viable options for the energy storage system for the Hyperloop.

Conclusion

The three energy-storage systems selected (LightSail, lithium-ion batteries, and lead-acid batteries) were reviewed using a Pugh Matrix. The Pugh Matrix assessed these options in the following categories: energy density, safety, cost, performance/efficiency, and reliability. By ranking these methods on a scale from 1-5 in each category, it was determined that the best energy storage system of the three analyzed for the Hyperloop is the lithium-ion battery.

Lithium-ion batteries have a very high efficiency, and it is unlikely that another storage option will have a significantly improved efficiency. It is acknowledged that although the lithium-ion battery may be the best of the three selected options, it may not be the right energy storage system for the Hyperloop. This is evident in the weight required in order to supply the capsule with sufficient power. Lithium-ion batteries weigh the least compared to lead-acid batteries and LightSail, but at an estimated 4,000 kg, they weigh about 2.5 times more than the 3,400 lb (1545 kg) suggested by Musk[1]. In addition, lithium-ion batteries are extremely expensive. At an estimated 20-year cost of $913 million, lithium-ion batteries cost three times more than lead-acid batteries and 65 times more than LightSail. This suggests that there may be another option better suited to powering the Hyperloop. The $913 million dollars required is the highest of the three options but it is considerably inexpensive compared to the current high speed rail budget of $68.4 billion[1].

Possible alternatives are molten salt batteries or hydrogen fuel cells as they may possess higher energy densities than lithium-ion batteries. This is particularly important as the battery weight is tied to the required weight of the power system. If these alternatives have comparable densities to lithium-ion batteries,

improved energy density may allow them to meet the suggested weight.

A limitation in the analysis that should be taken into future consideration is that varying the size and weight of the batteries, along with many other variables, will alter the power needed for the compressor. Thus, calculations for the power requirement should take these variations into consideration. This should considerably alter the conclusions drawn as to which energy storage system is ideal for the Hyperloop.

Appendix A - General Calculations

Energy Consumption of Compressor

325 kWh $\times$ 45min $\times$ 1 hr60 min=243.75 kWh (A1)

Appendix B - LightSail Calculations

Volume of the air

The work from LightSail can be modeled as an isothermal, reversible gas expansion.

dW=PdV

Pressure is too high to assume ideal gas conditions; the Redlich-Kwong takes intermolecular interactions that gases experience at high pressures; it is as follows:

P=RTVm-b-aTVm(Vm+b)(B1)

a and b are constants that vary depending on the gas. LightSail uses air, so corresponding constants for air will be used in the calculation [20].

0WdW=Vm1Vm2(RTVm-b-aTVm(Vm+b))ndVm(B2)

Vm1 and Vm2 are the molar volume at the start and end of the expansion, respectively. *n* is number of total moles. The expansion can be considered to have the following parameters:

T=300K R=82.05746cm3*atmK*mol η=0.70

a=15.65*106 (atm)(K12)(cm3mol)2 b=25.3cm3molP0=204.14 atm

Initial pressure is known as P0=204atm, so initial molar volume Vm1 can be calculated [21]. η is the efficiency of LightSail regarding work output.

P0=RTVm1-b-aTVm1Vm1+b (B3)

Vm1=256.5cm3

After specifying work needed, *W*, the following system of equations must be solved in a manner that minimizes volume V by choosing the appropriate value of number of moles n.

W/η=Vm1Vm2(RTVm-b-aTVm(Vm+b))ndVm (B4)

*V=Vm2*n*

The capsule needs 243.75 kilowatt-hours of work during its 45-minute period of operation, and solving for the volume gives

Vair=17.1m3

Volume of the Water

As LightSail uses hot water to give the gas the energy needed for expansion, this water could potentially take up some space as well.

dQ/η=mcdT

dQ is the change in heat, *dt* is the change in temperature, *m* is the mass, and *c* is the specific heat capacity of water. The hot temperature of LightSail is assumed about 550K and the cold temperature in a typical heat engine might be about 270K. With the heat capacity c=0.0116kWh/kg*K, the mass of the water needed can be calculated.

*m=243.75kWh/.70.00116kWhkg*K*550K-270K=750kg (B5)*

Transforming mass into volume, with density ρ=1000kg/m3

Vwater=mρ=750kg1000m3

Vwater=1.11m3

To find the combined volume of air and water, the two volumes are added together:

Vair+water=Vair+Vwater=17.1m3+1.11m3 (B6)

Vair+water=18.2m3

Volume of Piston

The maximum diameter of the piston is equivalent to the maximum height of the Hyperloop capsule, 1.10 m. Using the ASME guidelines for pressure vessels, the walls of the stainless steel piston must be 4.01 inches (0.102 m) thick [22]. The area of the piston is calculated:

Atot-Acenter=Apiston, A=πr2 (B7)

πDout24-πDin24=Apiston

π1.1024-π1.10-2.10224=.320 m2

The volume of the piston is equivalent to the length of the piston multiplied by the inner area:

VmaxAcenter=L (B8)

17.1π1.10-2.10224=27.1 m

If the piston walls are a uniform thickness along the length, the volume of steel required per piston is

*V=A*L=.320*27.1=8.67 m3 (B9)*

Using the density of steel, 8027 kg/m^3, the mass of steel required per piston is 48,800 kg steel [23].

Mass=8.67 m3×8027kgm3=69,600 kg (B10)

Appendix C - Lead Acid Battery Calculations

In order to produce 243.75 kWh of energy to the compressor, assuming that the lead acid batteries have a specific energy of 35 Wh/kg, can be depleted to no less than 40% of their capacity and the discharge efficiency of the batteries is 71%, the mass of lead acid batteries required is:

243.75 kWh × 1000 WhkWh × 1 kg35 Wh ×10.60 × 10.71=16,348 kg (C1)

Assuming an energy density of 70 Wh/L, the volume of the lead acid batteries required is:

243.75 kWh × 1000 WhkWh × 1 L70 Wh × 10.6 × 10.71=8,174 L (C2)

Appendix D - Lithium Ion Battery Calculations

In order to produced 243.75 kWh of energy to the compressor, assuming that the lithium ion batteries have a specific energy of 120 Wh/kg, can be depleted to no less than 40% of their capacity, and have a discharge efficiency of 85%, the mass of lithium ion batteries required is:

243.75 kWh × 1000 Wh1 kWh × 1 kg120 Wh × 10.60 × 10.85= 3,983 kg (D1)

Assuming an energy density of 230 Wh/L, the volume of the lithium ion batteries required is:

243.75 kWh × 1 L0.230 kWh × 10.85 × 10.60=2078 L (D2)

Appendix E - Cost of Energy Storage Systems

LightSail

With an estimated minimum piston mass of 48,800 kg of stainless steel, and a cost of $1,500 per tonne of stainless steel, the minimum estimated cost of LightSail is:

69,600 kgpiston × 1 tonne1000 kg × $1,500tonne=$104,400piston (E1)

It is assumed that the cost of the piston will comprise the majority of the cost for a LightSail module. If it is assumed that this cost is approximately 75% of the total cost, the estimated cost of a LightSail module is:

$104,400piston × 10.75= $97,600piston ≅$139,200piston (E2)

With an estimation of 100 capsules – which is notably much higher than the realistic number of capsules in commission at any given time – and a minimum lifetime of 20 years, the cost of LightSail is:

$140,00piston × 100 pistons=$14 million (E3)

Batteries

With a departure rate of one capsule every two minutes from both San Francisco and Los Angeles:

2 trips2 minutes × 1440 minutesday= 1440 tripsday= 525,600 tripsyear (E4)

After each trip, the battery needs to be recharged. The number of recharges during the lifetime of the battery varies with battery type.

Lithium-ion

With 1900 recharges during the lifetime of a lithium-ion battery:

525,600 rechargesyear × 1 battery1900 recharges= 277 batteriesyear (E5)

Given the power requirement of the battery (243.75 kWh) and the cost per Wh for lithium-ion batteries ($0.689/Wh), the cost of each battery is:

243.75 kWhbattery × $0.689Wh × 1000 WhkWh= $167,943.75battery ≅ $168,000battery (E6)

Thus, the annual cost of lithium-ion batteries is:

277 batteriesyear × $168,000battery= $46.536 millionyear ≅ $46.5 millionyear (E7)

Over a time span of twenty years (the assumed lifespan of LightSail), the estimated cost of using lithium-ion batteries is:

$46.5 millionyear × 20 years ≅$931 million (E8)

Lead-Acid

With 1200 recharges during the lifetime of a lithium-ion battery:

525,600 rechargesyear × 1 battery1200 recharges= 438 batteriesyear (E9)

Given the power requirement of the battery (243.75 kWh) and the cost per Wh for lithium-ion batteries ($0.15/Wh), the cost of each battery is:

243.75 kWhbattery × $0.15Wh × 1000 WhkWh= $36,562.5battery ≅ $36,600battery (E10)

Thus, the annual cost of lithium-ion batteries is:

438 batteriesyear × $36,000battery= $15.768 millionyear ≅ $15.8 millionyear (E11)

Over a time span of twenty years (the assumed lifespan of LightSail), the estimated cost of using lithium-ion batteries is:

$15.8 millionyear × 20 years ≅$316 million (E12)

Appendix G - MATLAB Code Used to Calculate Volume Air Required

```
clear all
clc
S=input('Desired work in kWh is =');
%Parameters
T=300;
a=15.65*10^6;b=25.3;
P=204.16;
R=82.05746;
W=3600000;
%Solving for Vm1
syms Vm X
Vm1=solve(P==R*T/(Vm-b)-a/(sqrt(T)*Vm*(Vm+b)));
Vm1=Vm1(1);
%Finding Smallest Volume
V=@(n) solve(int(R*T/(X-b)-a/(sqrt(T)*X*(X+b)),Vm1,X)*n-W*S/0.7)*n*.000001;
[N,fval]=fminsearch(V,S*152);
fprintf('The volume of the air after expansion will be %3.1f cubic meters\n',fval)
```

References

1. Musk, Elon. "Hyperloop Alpha." *Tesla Motors*. Tesla Motors, 12 Aug 2013. Web. <http://www.teslamotors.com/sites/ alpha.pdf[1] >
2. "Technology." *LightSail Energy*. LightSail Energy. Web. <http://www.lightsailenergy.com/[2] >.
3. Brain, Marshall. "How Lithium-ion Batteries Work." *HowStuffWorks*. Discovery Communications, 2013. Web. <http://electronics.howstuffworks.com/everyday-tech/lithium-ion-battery.htm[3] >.
4. "Lithium-based Batteries." *Battery University*. Cadex Electronics Inc., 2013. Web. <http://batteryuniversity.com/lear >.
5. Kassner, Michael. "Five Tips for Extending Lithium-ion Battery Life." *TechRepublic*. CBS Interactive, 2013. Web. <http://www.techrepublic.com/blog/five-apps/five-tips-for-extending-lithium-ion-battery-life/289[5] >.

[1] http://www.teslamotors.com/sites/default/files/blog_images/hyperloop-alpha.pdf
[2] http://www.lightsailenergy.com/
[3] http://electronics.howstuffworks.com/everyday-tech/lithium-ion-battery.htm
[4] http://batteryuniversity.com/learn/article/lithium_based_batteries
[5] http://www.techrepublic.com/blog/five-apps/five-tips-for-extending-lithium-ion-battery-life/289

6. "Can the Lead-acid Battery Compete in Modern Times?" *Battery University*. Cadex Electronics Inc., 2013. Web. <http://batteryuniversity.com/learn/article/can_the_lead_acid_battery_compete_in_modern_times[6] >.
7. Ultralife Corporation. *Li-Ion vs. Lead Acid*. Newark: Ultralife Corporation, 2013. Print. <ultralifecorporation.com/dov >.
8. Gage, Deborah. "Peter Thiel, Bill Gates, Khosla Get Behind Energy Storage Start-Up LightSail in $37M Deal." *The Wall Street Journal*. Dow Jones & Company, 5 Nov. 2012. Web. <http://blogs.wsj.com/venturecapita thiel-bill-gates-khosla-get-behind-energy-storage-start-up-lightsail-in-37m-deal/[8] >.
9. "Lead-acid Battery Safety." *University of Wisconsin System*. Board of Regents - University of Wisconsin System, 2010. Web. <http://www.uwsa.edu/ehs/occupational-safety/misc-safety/battery-safety/>.
10. Berman, Brad. "McKinsey: Lithium Ion Battery Prices to Reach $200 Per kWh by 2020." *Plugincars*. McKinsey, 16 July 2012. Web. <http://www.plugincars.com/lithium-ion-battery-prices-drop-160-kwh-2025-123193.html[9] >.
11. Evans, Keith. "The Future of Electric Vehicles: Setting the Record Straight on Lithium Availability." *Journal of Energy Security*. Institute for the Analysis of Global Security, 27 Aug. 2009. Web. <http://www.ensec.org/index.php?option=com_content&id=213:the-future-of-electric-vehicles-setting-the-record-straight-on-lithium-availability&catid=98:issuecontent0809&Itemid=349[10] >
12. Fehrenbacher, Katie. "Introducing an ultra low cost, long lasting battery made of water and blue dye" *GIGaom*. Alveo Energy, 4 Dec. 2012. Web. <http://gigaom.com/2012/12/04/introducing-an-ultra-low-cost-long-lasting-battery-made-of-water-and-blue-dye/[11] >
13. Stevens, J.W.,and G.P. Corey. "A Study Of Lead-acid Battery Efficiency Near Top-of-charge and the Impact On PV System Design." *25. Photovoltaic Solar Energy Conference, Washington, DC (United States)*, 13-17 May 1996.
14. Valøen, Lar Ole; Shoesmith, Mark I.; "The Effect of PHEV and HEV Cycles on Battery and Battery Pack Performance" Proceedings of the PHEV, 2007. Web. <http://www.pluginhighway.ca/PHEV2007/proceedings/Plu >
15.
16. Palo Alto Research Center. *High Energy Density Fuel Cell Systems: Up to 8 times higher than existing battery technologies.* Palo Alto: Palo Alto Research Center Inc, 2002. Print. <http://www.parc.com/content/attachme e_parc.pdf[13] >.
17. "What Is Energy Density." Web log post. FuelCellsforPower. Web. 2 Oct. 2013.<http://www.fuelcellsforpower.com/ind >
18. Narayanan, S. R. "Advances in Materials and System Technology for Portable Fuel Cells."*Indo-US Workshop*. NASA-Jet Propulsion Laboratory, Web. 02 Oct. 2013. <http://trs-new.jpl.nasa.gov/dspace/bitstream/2014 0619.pdf>.
19. Palo Alto Research Center. *High Energy Density Fuel Cell Systems: Up to 8 times Higher than Existing Battery Technologies*. Palo Alto, California: Palo Alto Research Center, 2010. Web. 2 Oct. 2013.<http://www.parc.com/content/attachments/fuelcells_arpa-e_parc.pdf[15] >
20. *The Sodium Nickel Chloride "Zebra" Battery*. Tech.: Meridian International Research, 2005. *Battery Technologies*. Web. <http://www.meridian-int-res.com/Projects/Zebra_Pages.pdf[16] >

[6]http://batteryuniversity.com/learn/article/can_the_lead_acid_battery_compete_in_modern_times
[7]http://ultralifecorporation.com/download/275/
[8]http://blogs.wsj.com/venturecapital/2012/11/05/peter-thiel-bill-gates-khosla-get-behind-energy-storage-start-up-lightsail-in-37m-deal/
[9]http://www.plugincars.com/lithium-ion-battery-prices-drop-160-kwh-2025-123193.html
[10]http://www.ensec.org/index.php?option=com_content&id=213:the-future-of-electric-vehicles-setting-the-record-straight-on-lithium-availability&catid=98:issuecontent0809&Itemid=349
[11]http://gigaom.com/2012/12/04/introducing-an-ultra-low-cost-long-lasting-battery-made-of-water-and-blue-dye/
[12]http://www.pluginhighway.ca/PHEV2007/proceedings/PluginHwy_PHEV2007_PaperReviewed_Valoen.pdf
[13]http://www.parc.com/content/attachments/fuelcells_arpa-e_parc.pdf
[14]http://www.fuelcellsforpower.com/index.php?option=com_content&view=article&id=51&Itemid=57
[15]http://www.parc.com/content/attachments/fuelcells_arpa-e_parc.pdf
[16]http://www.meridian-int-res.com/Projects/Zebra_Pages.pdf

21. "Green Car Congress: Sumitomo considering Marketing New Lower-temperature Molten-salt Electrolyte Battery to Automakers for EVs and Hybrids." *Green Car Congress: Sumitomo considering Marketing New Lower-temperature Molten-salt Electrolyte Battery to Automakers for EVs and Hybrids*. Web. 03 Oct. 2013.<http://www.greencarcongress.com/2011/11/sumitomo-2011111.html[17] >
22. Bambase, Manolito E., Jr. "Constants for Van Der Waals and Redlich-Kwong Equations" *Che 31. Introduction to Chemical Engineering*. N.p., 12 June 2010. Web. 23 Sept. 2013. <che31.weebly.com/uploads/3/7/4/3/37437 realgasconstants.pdf[18] >.
23. Wesoff, Eric, and Herman Trabish. "LightSail Gets $5.5M From Total, Thiel, Khosla, Gates for Compressed Air Energy Storage." *Greentech Media*. Greentech Media, Inc., 19 Feb. 2013. Web. <http://www.greentechmedia.com/articles/read/LightSail-Gets-5.5M-from-Total-Thiel-Khosla-Gates-for-Compressed-Air-En[19] >.
24. "Pressure Vessel Thickness Calculation." *CheCalc*. Checalc, n.d. Web. <http://checalc.com/calc/vesselThick.html[20] >
25. "Types 316 (S31600), 316L (S31603), 317 (S31700), 317L (S31703)." *Main Product Technical Information*. Shanghai Stal Precision Stainless Steel Co., LTD, 2013. Web.<http://www.stal.com.cn/pdffile/316316l317317l.pdf >.

[17]http://www.greencarcongress.com/2011/11/sumitomo-2011111.html
[18]http://che31.weebly.com/uploads/3/7/4/3/3743741/handout_e-realgasconstants.pdf
[19]http://www.greentechmedia.com/articles/read/LightSail-Gets-5.5M-from-Total-Thiel-Khosla-Gates-for-Compressed-Air-En
[20]http://checalc.com/calc/vesselThick.html
[21]http://www.stal.com.cn/pdffile/316316l317317l.pdf